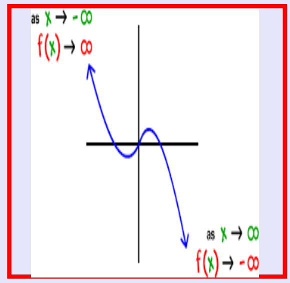
Math 245

Spring 2022

Lecture 36



Descarte's Rule of Signs:

Let $P(x)$ be a polynomial with real coefficients, in descending order and nonzero constant.

$$P(x) = 5x^4 + 3x^2 + 2x - 9$$

1) The number of positive real zeros either equals to the number of variations in signs or reduced by positive even integer.

2) First find $P(-x)$, the number of negative real zeros either equals to the number of variations in signs of $P(-x)$ or reduced by positive even integer.

$P(x) = 5x^4 + 3x^2 + 2x - 9$

1 Variation $\Rightarrow$ P(x) has 1 Positive Zero.

$P(x) = 5x^4 + 3x^2 + 2x - 9$

$P(-x) = 5(-x)^4 + 3(-x)^2 + 2(-x) - 9$

$= 5x^4 + 3x^2 - 2x - 9$

1 Variation in P(-x) $\Rightarrow$ P(x) has 1 Negative Zero.

$P(x) = 5x^4 + 3x^2 + 2x - 9$

At most 4 Zeros

Positive	Negative	Complex
1	1	2

$P(x) = x^4 - 6x^3 + 8x^2 + 2x - 1$

Polynomial, Real Coef., Nonzero constant, Descending order

Degree = 4 $\Rightarrow$ At most 4 Zeros

$P(x) = x^4 - 6x^3 + 8x^2 + 2x - 1$

1 2 3

3 Variations of Signs in P(x)

$\Rightarrow$ 3 positive or 1 positive Zero

$P(-x) = (-x)^4 - 6(-x)^3 + 8(-x)^2 + 2(-x) - 1$

$= x^4 + 6x^3 + 8x^2 - 2x - 1$

1

1 Variation in Signs of P(-x)

$\Rightarrow$ 1 Negative Zero

Must be all even

Sum on each row = degree of P(x)

Positive	Negative	Complex
3	1	0
1	1	2

$P(x) = -8x^4 + 3x^3 - 6x^2 + 5x - 7$

Polynomial, Descending order, real Coef., NonZero Constant

Degree $= 4 \Rightarrow$ At most 4 Zeros

$P(x) = -8x^4 + 3x^3 - 6x^2 + 5x - 7$

1 2 3 4

4 Variations in Signs of $P(x) \Rightarrow 4, 2,$ or 0 Positive Zeros.

$P(-x) = -8(-x)^4 + 3(-x)^3 - 6(-x)^2 + 5(-x) - 7$

$= -8x^4 - 3x^3 - 6x^2 - 5x - 7$

0 Variation in Signs of $P(-x) \Rightarrow 0$ negative Zeros

At most 4 Zeros

Pos.	Neg.	Complex
4	0	0
2	0	2
0	0	4

Sum on each row = degree

All even

$P(x) = -2x^5 + 10x^4 - 6x^3 + 8x^2 - x + 1$

Polynomial, Real Coef., Non Zero constant, Descending order

At most 5 Zeros

$P(x) = -2x^5 + 10x^4 - 6x^3 + 8x^2 - x + 1$

1 2 3 4 5

Positive Zeros $\Rightarrow$ 5, 3, or 1

$P(-x) = -2(-x)^5 + 10(-x)^4 - 6(-x)^3 + 8(-x)^2 - (-x) + 1$

$= 2x^5 + 10x^4 + 6x^3 + 8x^2 + x + 1$

All even

Negative Zeros $\Rightarrow$ 0

Pos.	Neg.	Complex
5	0	0
3	0	2
1	0	4

Sum on each row = degree

$P(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

Poly nomial, Real Coef., NonZero Constant, Descending order

At most 4 Zeros

$P(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

1 2 3

Positive Zeros ⇒ 3 or 1

$P(-x) = 2(-x)^4 - (-x)^3 + 7(-x)^2 - 4(-x) - 4$

$= 2x^4 + x^3 + 7x^2 + 4x - 4$

1

Negative Zeros ⇒ 1

All even

Sum of each row = deg.

Pos.	Neg.	Complex
3	1	0
1	1	2

Is 1 a Zero?

1	2	-1	7	-4	-4
		2	1	8	4
	2	1	8	4	0

Remainder = 0

1 is a Zero.

Is $\frac{-1}{2}$ a Zero?

$\frac{-1}{2}$	2	1	8	4
		-1	0	-4
	2	0	8	0

Rem = 0

$\frac{-1}{2}$ is a Zero.

$P(x) = (x-1)\left(x - \frac{-1}{2}\right)(2x^2 + 8)$

$= (x-1)\left(x + \frac{1}{2}\right) \cdot 2(x^2+4) = (x-1)(2x+1)(x^2+4)$

$x^2 + 4 = 0$

2 Complex Zeros → $\boxed{x = \pm 2i}$